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- USSR -

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THE MOTION OF A THIN BLUNTED CONE WITH A SMALL ANGLE OF ATTACK AT A HIGH SUPERSONIC SPEED

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/Following is a translation of an article by V. V. Junev, I. N. Murzinov, and O. N. Ostapovich in the Russian-language periodical Izvestiya Akademii Nauk SSSR, OTN, Mekhanika i Mashinostroveniye (News of the Academy of Sciences of the USSR, Division of Technical Sciences, Moscow, No. 3, May-June 1960, Pages 121-125./

The effect of bluntness /zatupleniya/ on the distribution of pressure on the generatrix of thin blunted cones with a small angle of attack α during great numbers of M is examined below. A qualitative illustration of flow is constructed on the basis of approximate results.

An analogous problem without an angle of attack was investigated in studies /1, 2/. The results of experimental studies are presented. An approximate law of similarity is established which determines the distribution of pressure on the generatrix of the cones specified.

1. We shall introduce a cylindrical system of coordinates xr_0, yr_0 and ϕ in which coordinate x is read from the locus of the transition of bluntness in the conical part along the axis of the body; y is the normal /normal'na/ of the axis; ϕ is the angle of deflection of the meridional surface, where $\phi = 0^0$ corresponds to the shaded /aztennenov/ side; r_0 is the radius of the midship section /midel'/ of the bluntness. In the system of coordinates selected, the velocity of the incident flow, with a precision up to α^2, will have components U_∞, $\alpha U_\infty \cos\phi$ -- $\alpha U_\infty \sin\phi$ respectively.

Let uU_∞, vU_∞, and wU_∞ be the components of the velocity along axes x, y, and Θ; $p\rho_\infty U_\infty^2$, $\rho\rho_\infty$, εU_∞^2, iU_∞^2 be the pressure, density, external energy and enthalpy of one /raza/; Rr_0 be the distance from the axis of the body to the shock wave; and rr_0 be the distance from the axis of the body to the surface of the cone. Index ∞ belongs to the dimensional parameters of the incident flow.

We shall assume $u \approx 1$ in the perturbed area. This is valid everywhere, beginning with the small (of the order r_0) withdrawal from the tip, with the exception of the high-entropic boundary layer, the effect of which is negligibly small /2/ in the

distribution of pressure in the case of a perfect gas $\varkappa = 1.4$ (which is assumed below).

Under these conditions, the equations of the conservation of mass, impulse and energy for a volume bounded by surfaces of a shock wave, a body and two proximate meridional surfaces, will have the following appearance, respectively, in integral form

$$\int_r^R \rho y dy = K(\psi) + \int_0^x \left(R \frac{\partial R}{\partial x} - \varkappa \cos \varphi R - \frac{\partial}{\partial \varphi} \int_r^R \rho w \, dy\right) dx \tag{1.1}$$

$$\int_r^R \rho v y dy = I(\psi) + \int_0^x \left(p_w r - \frac{R}{\gamma M^2} + \varkappa \cos \varphi R \frac{\partial R}{\partial x} + \int_r^R p \, dy - \frac{\partial}{\partial \psi} \int_r^R \rho w v dy\right) dx$$

$$\int_r^R \left(\frac{p}{\gamma - 1} + \rho \frac{v^2}{2}\right) y dy = E(\psi) + \frac{R^2}{2\gamma(\gamma - 1) M^2} + \tag{1.2}$$

$$+ \int_0^x \left[p_w r \frac{dr}{dx} - \varkappa \frac{R}{(\gamma - 1) M^2} \cos \varphi - \frac{\partial}{\partial \varphi} \right. \tag{1.3}$$

$$\left. \int_r^R p w \, dy - \frac{\partial}{\partial \varphi} \int_r^R \left(\frac{p}{\gamma - 1} + \rho \frac{v^2}{2}\right) w dy \right] dx$$

Here $K(\phi)$ is the dimensionless flow of mass between the body and the shock wave when $x = 0$; $I(\phi)$ and $E(\phi)$ are dimensionless impulse along axis y and energy which are imparted to the gas by the bluntness; and p_w is the pressure on the surface of the body.

The magnitudes $K(\phi)$, $I(\phi)$ and $E(\phi)$ can be determined by examining the approximately concrete conditions of flow around a blunted surface with a departing shock wave.

We shall examine two forms of bluntness -- as part of a sphere which is tangent with the subsequent surface of the body when $x = 0$, and as a flat end plane.

In the first case, let γ be the angle between the radius of a sphere and the axis of the cone, and γ_1 be the angle between the same radius and the direction at the critical point.

We shall assume that the pressure on the sphere is distributed according to Newtonian theory, then

$$\gamma_1 = \gamma + \alpha \cos \varphi \,, \quad p = \cos^2 \gamma_1 = \cos^2 \gamma -$$
$$- 2 \alpha \sin \gamma \cos \gamma \cos \varphi$$

The other parameters of flow can be described by the usual formulas of gas dynamics by means of the pressure which is assumed constant along the normal to the surface of the body and the tilt angle of the shock wave (also assumed to be spherical) at the point of intersection of its linear flow under consideration. In setting up the equations of balance, it is necessary to consider in this case that the flow lines intersect the meridional surface ϕ at an angle equal to $\alpha \sin \phi / \sin \gamma$ where $\gamma \gg \alpha$. Consequently, there is a transfer of mass, impulse and energy through the boundaries of the area under consideration between two proximate meridional areas.

In the second case, the scheme based on experimental observations was used. In accordance with the scheme, the shock wave practically does not change its form in relation to the direction of the incident flow and the distance of the sonic point to the axis is close to the radius of the end plane for small α's departing from the flat end plane. It was assumed that the sonic line on the meridional surface is the line segment between the sonic point in the shock wave and the angular point of the end plane and that the parameters of flow, known at the ends of the wave when $\alpha = 0$, are determined in both cases by the data of study [3]. The assumptions made are sufficient to determine the magnitudes $K(\phi)$, $I(\phi)$ and $E(\phi)$ which, however they are ascertained, have the following form for small α's.

$$K(\varphi) = K_0 + \alpha \cos \varphi \, K_1, \qquad I(\varphi) = I_0 + \alpha \cos \varphi \, I_1 \qquad (1.4)$$

$$E(\varphi) = E_0 + \alpha \cos \varphi \, E_1$$

Here K_0, I_0 and E_0 are magnitudes corresponding to the flow [obtekaniye] of a body without an angle of attack; K_1, I_1 and E_1 are constant.

These magnitudes have the following values when $M \gg 1$:

for a sphere

$$K_0 = \frac{R_0^2(0)}{2}, \qquad I_0 = 0.3, \qquad E_0 = 0.46,$$

$$K_1 = 0.78, \qquad I_1 = -0.43, \; E_1 = 0.06$$

for a flat end plane

$$K_0 = \frac{R_0^2(0)}{2}, \qquad I_0 = 0.45, \qquad E_0 = 0.9,$$

$$K_1 = 1.67, \qquad I_1 = 0.68, \qquad E_1 = 1.4$$

Let us assume, as in studies [5, 2], that the pressure in the perturbed layer is $p = p_w(x, \phi)$ and that the velocity is equal to the velocity after the compression jump /za skachkom uplotneniya/ for a given x and ϕ.

Equation (1.1) through (1.3) and condition (1.4) permit a solution to be found with precision up to α^2 in the form

$$v = v_0(x) + \alpha \cos \varphi \; v_1(x), \qquad w = \alpha \sin \varphi \; w_1(x) \tag{1.5}$$

$$p = p_0(x) + \alpha \cos \varphi \, p_1(x), \quad \rho = \rho_0(x,y) + \alpha \cos \varphi \rho_1(x,y)$$

$$R = R_0(x) + \alpha \cos \varphi \, R_1$$

Here the functions with index 0 represent the solution to the problem of the passage of a cone without an angle of attack [2] and functions v_1, w_1 and R_1 are related in the equations

$$v_1 = 1 - \frac{2}{\varkappa + 1}(1 - R_1{'})\left(1 + \frac{1}{R_0{'}^2 M^2}\right) \tag{1.6}$$

$$w_1 = -1 + \frac{2}{\varkappa + 1} \frac{R_0{'} R_1}{R_0}\left(1 - \frac{1}{R_0{'}^2 M^2}\right)$$

Substituting (1.5) in (1.1) through (1.3), setting zero equal to the sum of the terms of the order α and excluding integrals containing ρ_1 from them, we obtain a system of equations for R_1 and p_1

$$
\begin{aligned}
R_1' = {} & \frac{(\varkappa + 1)\, R_o'^2 M^2}{R_o^2 \,(1 + R_o'^2 M^2)} \left\{ \frac{R_o^2}{\varkappa + 1} \left(1 + \frac{1}{R_o'^2 M^2}\right) + \int_0^x R_o p_1 \, dx + \right. \\
& + \int_0^x \left(p_s - \frac{1}{\varkappa M^2}\right) R_1 \, dx + v_o \int_0^x R_o dx - \int_0^x v_o w_1 \int_r^{R_o} \rho_o dy \, dx + \\
& + v_o \int_0^x w_1 \int_r^{R_o} \rho_o \, dy \, dx - v_o R_o R_1 + v_o \left[R_o(0)\, R_1(0) - K_1\right] + \\
& \left. + I_1 - \tfrac{1}{2} R_o^2(0) \right\}
\end{aligned}
\tag{1.7}
$$

$$
\begin{aligned}
p_1 = {} & \frac{2(\varkappa - 1)}{R_o^2 - r^2} \left\{ \frac{v_o^2}{2} \left[R_o R_1 - R_o(0)\, R_1(0)\right] - \frac{v_o^2}{2} \int_0^x w_1 \right. \\
& \int_r^{R_o} \rho_o \, dy \, dx + v_o \int_0^x v_o w_1 \int_r^{R_o} \rho_o \, dy \, dx - \int_0^x \frac{v_o^2}{2} w_1 \int_r^{R_o} \rho_o \, dy \, dx - \\
& - \frac{v_o^2}{2} \int_0^x R_o \, dx - v_o \int_0^x p_1 R_o dx - v_o \int_0^x R_1 \left(p_s - \frac{1}{\varkappa M^2}\right) dx + \\
& + \int_0^x p_1 \, r r' \, dx - \frac{\varkappa}{\varkappa - 1} \int_0^x p_o \, w_1 \,(R_o - r) \, dx - \\
& - \int_0^x \frac{R_o}{(\varkappa - 1) M^2} dx - \frac{R_o R_1}{\varkappa - 1} \left(p_s - \frac{1}{\varkappa M^2}\right) + \frac{v_o^2}{2} (K_1 - R_o^2) + \\
& \left. + v_o \left[\frac{R_o^2(0)}{2} - I_1\right] + E_1 \right\}
\end{aligned}
$$

$$
p_s = \frac{2}{\varkappa + 1} \left(R_o'^2 - \frac{\varkappa - 1}{2 \varkappa M^2} \right)
$$

Since the difference of $R_0 - r$ is small and the density ρ_0 is small close to the surface of the body, we can assume in (1.7)

$$\int_r^{R_0} \rho_0 \, dy \approx \frac{1}{R_0} \int_r^{R_0} \rho_0 y dy = \frac{R_0}{2}$$

To solve equations (1.7), the magnitudes p_0 (0) and R_1 (0) are necessary, for which the supplementary relationship for $x \approx 0$ is required. However, the calculations carried out for the case of blunting a spherical form in the different approaches to this relationship (dotted line in Figure 1) showed that the solution even for $x \geqslant 2$ practically does not depend on the values of p_1 (0) and R_1 (0).

2. Figure 1 presents calculations of the value p_1 (x) when $M = \infty$ for cones with a semispan /polurastvor/ angle $\Theta_0 = 10^0$ and with blunting in the form of a sphere (curve 1) and the flat end plane (curve 2). For large values of x, these solutions are in good agreement with the solution for a sharp cone /4, 5/.

In Figures 2 and 3, the curves of pressure distribution for $\alpha = 0.5^0$ in accordance with the generatrix of $\phi = 0^0$ and $\phi = 180^0$ of the lines calculated were constructed (the unbroken lines: Figure 2 -- sphere; Figure 3 -- end plane) on coordinates

$$Y = \frac{p}{\beta^2}, \qquad \zeta = \sqrt{\frac{2}{C_x}} \, x \beta^2 \tag{2.1}$$

$$(\beta = \Theta_0 + \alpha \cos \varphi)$$

Here, C_x is the resistance coefficient of bluntness.

The same figures present curves of the pressure distribution for $\alpha = 0$ on cones with blunting in the form of a sphere and flat end planes (dotted line), calculated by the method of study /2/ for $\Theta_0 = 7.5^0$ and 10^0. As is clear, all the curves mentioned turn out to be proximate to some single curve $Y = f(\zeta)$ in the coordinates indicated.

Note that for $\alpha = 0.5^0$ the pressure differs by 15% from the pressure for $\alpha = 0$. From this point of view the angles of attack $\alpha > 0.5^0$, generally speaking, are still not small for the case under consideration.

The coordinates (2.1) for $\alpha = 0$ coincide with the coordinates of similarity of G. G. Chernyy /1/. For $M\Theta_0 \gg 1$, $p \approx \beta^2$ on a thin pointed cone, therefore, the most natural generalization of the coordinates in the case of $M\Theta_0 \gg 1$ is the following

$$Y = \frac{p - \frac{1}{\gamma M^2}}{p_k - \frac{1}{\gamma M^2}} \qquad (2.2)$$

$$\zeta = \sqrt{\frac{2}{C_x}} \qquad \left(p_k - \frac{1}{\gamma M^2} \right)$$

where p_k is the pressure on the similar generatrix of the pointed cone for these same Θ_o and α. For $M\Theta_o \gg 1$, the coordinates (2.2) coincide with (2.1).

Figure 3 presents, in the coordinates of (2.2), the results of an experimental study of flow for $M = 6$ and $\alpha = 4^o$, a cone with $\Theta_o = 10^o$ blunted on the flat end plane and also data for study [6] for $M = 6.85$ and $\alpha = 0^o$. As may be seen, all points for different ϕ and χ also lie close to a single curve.

The following approximation law of similarity can be formulated on the basis of the results presented.

For the flow of thin blunted cones with small angles of attack at hypersonic speeds, the distribution of pressure along the generatrix is determined by their local angle of attack and conform to the law of similarity in the form $Y = f(\zeta)$. Consequently, this distribution coincides with the distribution of pressure on the cone passing without an angle of attack with this same bluntness and a semispan angle equal to the local angle of attack of a given generatrix.

Strictly speaking, the equation $Y = f(\zeta)$ on the coordinates of (2.2) do not quite correspond to the formulation of this law of similarity since the magnitude $p_k (\Theta_o, \alpha, \phi, M)$ generally speaking do not coincide with the pressure when $\chi = 0$ on a pointed cone with a semispan angle equal to the local angle of attack of the generatrix for the data Θ_o, α, ϕ, M. However, this difference is unimportant when $M\Theta_o \sim 1$ and disappears when $M\Theta_o \gg 1$.

The law indicated is the general law of similarity, defined by G. G. Chernyy [1] for the case of cones passing without an angle of attack.

Figure 2 and 3 present data for $\Theta_o = 15^o$, which accords somewhat less well with the remaining data. This is explained apparently by the limited nature of the law of similarity [1]. It must be noted, however, that in absolute values, the pressures on cones $\Theta_o = 7.5^o$ and $\Theta_o = 15^o$ differ by four times.

Submitted 4-11-1960

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FIGURES

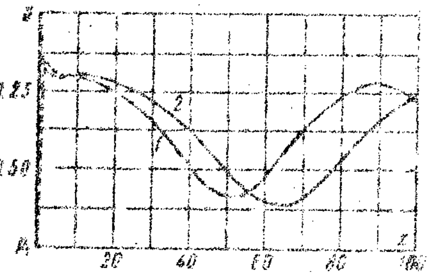

Figure 1.

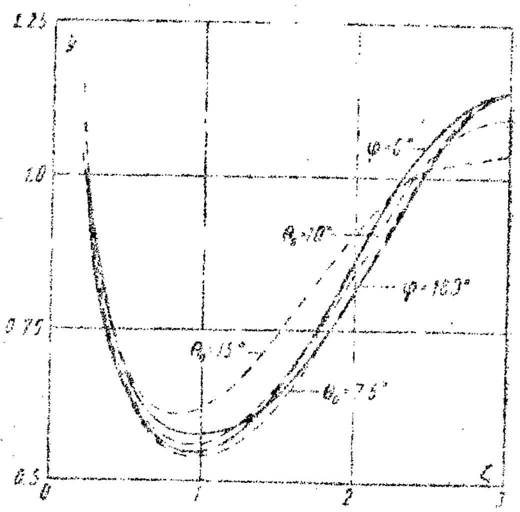

Figure 2.

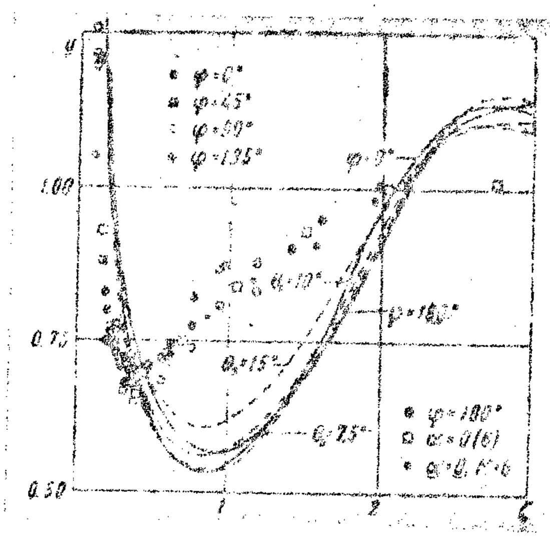

Figure 3.

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